

# Statistical entropy of a stationary dilaton black hole from the Cardy formula

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With Carlip's boundary conditions, a standard Virasoro subalgebra with a corresponding central charge for a stationary dilaton black hole obtained in the low-energy effective field theory describing string is constructed at a Killing horizon. The statistical entropy of a stationary dilaton black hole yielded by the standard Cardy formula agrees with its Bekenstein-Hawking entropy only if we take the period  $T$  of function  $v$  as the periodicity of the Euclidean black hole. On the other hand, if we consider a first-order quantum correction then the entropy contains a logarithmic term with a factor  $-\frac{1}{2}$ , which is different from that of Kaul and Majumdar,  $-\frac{3}{2}$ . We also show that the discrepancy is not just for the dilaton black hole, but for any one whose corresponding central charge takes the form  $c/12 = (A_H/8\pi G)(2\pi/\kappa T)$ .

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## I. INTRODUCTION

Much effort has been concentrated on the statistical mechanical description of the Bekenstein-Hawking black hole entropy [1–3] in terms of microscopic states both in string theory [4] and in “quantum geometry” [5]. Strominger [6] calculated the entropy of black holes whose near-horizon geometry is locally  $AdS_3$  from the asymptotic growth of states. Carlip [7,8] derived the central extension of the constraint algebra of general relativity by using the Brown-Henneaux-Strominger approach [6] and manifestly covariant phase space methods [9–12]. He found that a natural set of boundary conditions on the (local) Killing horizon leads to a Virasoro subalgebra with a calculable central charge and the standard Cardy formula gives the Bekenstein-Hawking entropies of some black holes. Those works show a suggestion that the asymptotic behavior of the density of states may be determined by the algebra of diffeomorphism at the horizon. Solodukhin [13] obtained the same result by an analysis of the Liouville theory near the horizon obtained from dimensional reduction of Einstein gravity. Das, Ghosh, and Mitra [14] studied the statistical entropy of a Schwarzschild black string in five dimensions by counting the black string states which form a representation of the near-horizon conformal symmetry with a central charge. Recently, we [15] extended Carlip's investigation [8] for the vacuum case to a case including a cosmological term and electromagnetic fields and calculated the statistical entropies of the Kerr-Newman black hole and the Kerr-Newman-AdS black hole by using standard Cardy formula.

On the other hand, the quantum correction to entropy of the black hole is an interesting topic [16–24]. Recently, Kaul

and Majumdar [16] computed the lowest order corrections to the Bekenstein-Hawking entropy in a particular formulation [25] of the “quantum geometry” program of Ashtekar *et al.* They showed that the leading corrections is a logarithmic term; i.e., the entropy is

$$S \sim \frac{A_H}{4} - \frac{3}{2} \ln \frac{A_H}{4} + \text{const} + \dots, \quad (1.1)$$

where  $A_H$  is the event horizon area. Carlip [17] also calculated the quantum corrections to black hole entropy by the Cardy formula and found that the entropy can be expressed as

$$S \sim S_0 - \frac{3}{2} \ln S_0 + \ln c + \text{const} + \dots, \quad (1.2)$$

where  $S_0$  is standard Bekenstein-Hawking entropy and  $c$  is a central charge of a Virasoro subalgebra. Carlip pointed out that if the central charge is the sense of being independent of the horizon area (Carlip thinks that this can be done by adjust the periodicity  $\beta$  [17]), then the factor of  $-3/2$  in logarithmic term will always appear.

We all know that four dimensional dilaton charged black hole obtained in the low-energy effective field theory describing strings have qualitatively different properties from those that appear in the ordinary Einstein gravity. Therefore, it is worth to investigate whether or not the Carlip's conclusion (the asymptotic behavior of the density of states may be determined by the algebra of diffeomorphism at horizon) and Kaul and Majumdar's result (the leading corrections to the entropy is a logarithm of the horizon area with a factor  $-3/2$ ) are valid for the static and stationary dilaton black hole.

We begin in Sec. II by using the covariant phase techniques to extend Carlip's investigation [8] for vacuum case  $L_{a_1 a_2 \dots a_n} = (1/16\pi G) \epsilon_{a_1 a_2 \dots a_n} R$  to a case for gravity

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coupled to a Maxwell field and a dilaton, i.e., the Lagrangian n-form is described by  $\mathbf{L}_{a_1 a_2 \dots a_n} = \epsilon_{a_1 a_2 \dots a_n} [R - 2(\nabla\phi)^2 - e^{-2\alpha\phi} F^2]$ . A constraint algebra is obtained. In Sec. III, the standard Virasoro subalgebra with corresponding central charges is constructed for stationary dilation black hole. The statistical entropy of the black hole is then calculated by using standard Cardy formula. In Sec. IV, a new Cardy formula is obtained and then the first-order quantum correction to the entropy is studied. The last section devotes to discussion and summary.

## II. ALGEBRA OF DIFFEOMORPHISM ON THE KILLING HORIZON

Let  $\xi^a$  be any smooth vector field on a spacetime manifold  $\mathbf{M}$ , i.e.,  $\xi^a$  is the infinitesimal generator of a diffeomorphism. Lee, Wald, and Iyer [9–12] showed that the Lagrangian  $\mathbf{L}$ , equation of motion n-form  $\mathbf{E}$ , symplectic potential (n-1)-form  $\Theta$ , Noether current (n-1)-form  $\mathbf{J}$ , and Noether charge (n-2)-form  $\mathbf{Q}$  satisfy following relations

$$\delta\mathbf{L} = \mathbf{E}\delta\phi + d\Theta, \quad (2.1)$$

$$\mathbf{J}[\xi] = \Theta[\phi, \mathcal{L}_\xi\phi] - \xi \cdot \mathbf{L}, \quad (2.2)$$

$$\mathbf{J} = d\mathbf{Q}, \quad (2.3)$$

Here and hereafter the ‘‘central dot’’ denotes the contraction of the vector field  $\xi^a$  into the first index of the differential form. Hamilton’s equation of motion is given by

$$\delta H[\xi] = \int_C \omega[\phi, \delta\phi, \mathcal{L}_\xi\phi] = \int_C [\delta\mathbf{J}[\xi] - d(\xi \cdot \Theta[\phi, \delta\phi])]. \quad (2.4)$$

By using Eq. (2.3) and defining a (n-1)-form  $\mathbf{B}$  as

$$\delta \int_{\partial C} \xi \cdot \mathbf{B}[\phi] = \int_{\partial C} \xi \cdot \Theta[\phi, \delta\phi], \quad (2.5)$$

the Hamiltonian can be expressed as [8]

$$H[\xi] = \int_{\partial C} (\mathbf{Q}[\xi] - \xi \cdot \mathbf{B}[\phi]). \quad (2.6)$$

The Poisson bracket forms a standard ‘‘surface deformation algebra’’ [26,8]

$$\{H[\xi_1], H[\xi_2]\} = H[\{\xi_1, \xi_2\}] + K[\xi_1, \xi_2], \quad (2.7)$$

where the central term  $K[\xi_1, \xi_2]$  depends on the dynamical fields only through their boundary values.

The four dimensional low-energy Lagrangian obtained from string theory is

$$\mathbf{L}_{abcd} = \epsilon_{abcd} [R - 2(\nabla\phi)^2 - e^{-2\alpha\phi} F^2], \quad (2.8)$$

where  $\epsilon_{abcd}$  is the volume element,  $\phi$  is the dilaton scalar field,  $F_{ab}$  is the Maxwell field associated with a  $U(1)$  subgroup of  $E_8 \times E_8$  or  $Spin(32)/Z_2$ , and  $\alpha$  is a free parameter which governs the strength of the coupling of the dilaton to

the Maxwell field. The reason we set the remaining gauge fields and antisymmetric tensor field  $H_{\mu\nu\rho}$  to zero is that the metrics of stationary and static dilaton black holes are almost obtained from the Lagrangian (2.8). We know from Lagrangian (2.8) that the equations of motion  $\mathbf{E}$  for dynamical fields  $A_\mu$ ,  $\phi$ , and  $g_{\mu\nu}$  can be respectively given by

$$\nabla_\mu (e^{-2\alpha\phi} F^{\mu\nu}) = 0, \quad (2.9)$$

$$\nabla^2\phi + \frac{1}{2}e^{-2\alpha\phi} F_{\mu\nu} F^{\mu\nu} = 0, \quad (2.10)$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 2\nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}(\nabla\phi)^2 + 2e^{-2\alpha\phi}F_{\beta\nu}F^\beta_\mu - \frac{1}{2}g_{\mu\nu}e^{-2\alpha\phi}F_{\mu\nu}F^{\mu\nu}. \quad (2.11)$$

The symplectic potential (n-1)-form is

$$\Theta_{bcd}[g, \mathcal{L}_\xi g] = 4\epsilon_{abcd} \left\{ \frac{1}{2}(\nabla_e \nabla^{[e} \xi^{a]} + R^a_e \xi^e) - \xi^e \nabla_e \phi \nabla^a \phi - e^{-2\alpha\phi} F^{af} [F_{ef} \xi^e + (\xi^e A_e)_{;f}] \right\}. \quad (2.12)$$

From Eqs. (2.2) and (2.12) we have

$$\begin{aligned} \mathbf{J}_{bcd} &= 2\epsilon_{abcd} \left\{ \nabla_e \nabla^{[e} \xi^{a]} - 2e^{-2\alpha\phi} F^{af} (\xi^e A_e)_{;f} \right. \\ &\quad \left. + \left[ R^a_e - \frac{1}{2} \delta^a_e R - 2\nabla_e \phi \nabla^a \phi + \delta^a_e (\nabla\phi)^2 - 2e^{-2\alpha\phi} F^{af} F_{ef} + \frac{1}{2} \delta^a_e e^{-2\alpha\phi} F^2 \right] \xi^e \right\} \\ &= 2\epsilon_{abcd} [\nabla_e \nabla^{[e} \xi^{a]} - 2e^{-2\alpha\phi} F^{af} (\xi^e A_e)_{;f}] \\ &= 2\epsilon_{abcd} [\nabla_e \nabla^{[e} \xi^{a]} + 4\nabla_f (e^{-2\alpha\phi} \nabla^{[f} A^{a]} A_e \xi^e)], \end{aligned} \quad (2.13)$$

in the second and third lines, we used the equations of motion (2.11) and (2.9). Equations (2.3) and (2.13) show that

$$\mathbf{Q}_{cd} = -\epsilon_{abcd} [\nabla^a \xi^b + 4e^{-2\alpha\phi} A_e \xi^e \nabla^a A^b]. \quad (2.14)$$

For a stationary dilaton black hole, the dilaton scalar field, the electromagnetic potential  $A_a$ , and the Killing vector can be respectively expressed as

$$\phi = \phi(r, \theta), \quad (2.15)$$

$$A_a = (A_0(r, \theta), A_1(r, \theta), A_2(r, \theta), A_3(r, \theta)), \quad (2.16)$$

$$\chi_H^a = \chi_H^{(t)} + \chi_H^{(\varphi)} = (1, 0, 0, \Omega_H), \quad (2.17)$$

where the vector  $\chi_H^{(t)}$  corresponds to time translation invariance,  $\chi_H^{(\varphi)}$  to rotational symmetry, and  $\Omega_H = -(g_{t\varphi}/g_{\varphi\varphi})_H$  is the angular velocity of the black hole.

As Carlip did in Ref. [8] we define a ‘‘stretched horizon’’  $\chi^2 = \epsilon$ , where  $\chi^2 = g_{ab} \chi^a \chi^b$ ,  $\chi^a$  is a Killing vector. The result of the computation will be evaluated at the event horizon of the black hole by taking  $\epsilon$  to zero. Near the stretched

horizon, one can introduce a vector orthogonal to the orbit of  $\chi^a$  by  $\nabla_a \chi^2 = -2\kappa \rho_a$ , where  $\kappa$  is the surface gravity. The vector  $\rho^a$  satisfies conditions

$$\begin{aligned} \chi^a \rho_a &= -\frac{1}{\kappa} \chi^a \chi^b \nabla_a \chi_b = 0 \quad \text{everywhere,} \\ \rho^a &\rightarrow \chi^a \quad \text{at the horizon.} \end{aligned} \quad (2.18)$$

To preserve ‘‘asymptotic’’ structure at horizon, we impose Carlip’s boundary conditions [8]

$$\begin{aligned} \delta \chi^2 &= 0, \quad \chi^a t^b \delta g_{ab} = 0, \quad \delta \rho_a = -\frac{1}{2\kappa} \nabla_a (\delta \chi^2) = 0, \\ &\quad \text{at } \chi^2 = 0, \end{aligned} \quad (2.19)$$

where  $t^a$  is a any unit spacelike vector tangent to boundary  $\partial \mathbf{M}$  of the spacetime  $\mathbf{M}$ . And the infinitesimal generator of a diffeomorphism is taken as

$$\xi^a = \mathcal{R} \rho^a + \mathcal{T} \chi^a, \quad (2.20)$$

where functions  $\mathcal{R}$  and  $\mathcal{T}$  obey the relations [8]

$$\mathcal{R} = \frac{1}{\kappa} \frac{\chi^2}{\rho^2} \chi^a \nabla_a \mathcal{T}, \quad \text{everywhere}$$

$$\rho^a \nabla_a \mathcal{T} = 0, \quad \text{at the horizon.} \quad (2.21)$$

For a one-parameter group of diffeomorphism such that  $D\mathcal{T}_\alpha = \lambda_\alpha \mathcal{T}_\alpha$ , ( $D \equiv \chi^a \partial_a$ ), one introduces an orthogonality relation [8]

$$\int_{\partial C} \hat{\epsilon} \mathcal{T}_\alpha \mathcal{T}_\beta \sim \delta_{\alpha+\beta}. \quad (2.22)$$

The technical role of the condition (2.22) is to guarantee the existence of generators  $H[\xi]$ . By using the other future-directed null normal vector  $N^a = k^a - \alpha \chi^a - t^a$ , with  $k^a = -1/\chi^2 (\chi^a - |\chi|/\rho \rho^a)$  and a normalization  $N_a \chi^a = -1$ , the volume element can be expressed as

$$\epsilon_{abcd} = \hat{\epsilon}_{cd} (\chi_a N_b - \chi_b N_a) + \dots, \quad (2.23)$$

the omitted terms do not contribute to the integral.

From the right hand of Eq. (2.5)

$$\begin{aligned} \int_{\partial C} \xi^b \Theta_{bcd} &= 4 \int_{\partial C} \epsilon_{abcd} \xi^a \left\{ \frac{1}{2} (\nabla_e \nabla^{[e} \xi^{b]} + R_e^b \xi^e) - \xi^e \nabla_e \phi \nabla^b \right. \\ &\quad \left. - e^{-2\alpha\phi} F^{fb} [F_{ef} \xi^e + (\xi^e A_e)_{;f}] \right\}, \end{aligned} \quad (2.24)$$

we know that the first two terms in the right hand of Eq. (2.24) can be treated as Carlip did in Ref. [8]. At the horizon, by using Eqs. (2.15), (2.16), (2.17), and (2.19)–(2.23) we obtain

$$\int_{\partial C} \epsilon_{abcd} \xi_2^a \xi_1^e \nabla^b \phi \nabla_e \phi = 0, \quad (2.25)$$

and

$$\begin{aligned} &\int_{\partial C} \epsilon_{abcd} e^{-2\alpha\phi} \xi^a F^{bf} [F_{ef} \xi^e + (\xi^e A_e)_{;f}] \\ &= \int_{\partial C} \epsilon_{abcd} e^{-2\alpha\phi} \xi^a F^{bf} \delta_\xi A_f \\ &= \int_{\partial C} \hat{\epsilon}_{cd} e^{-2\alpha\phi} \left[ \frac{|\chi|}{\rho} \mathcal{T} \rho_b + \left( \frac{\rho}{|\chi|} + t \cdot \rho \right) \mathcal{R} \chi_b \right] F^{bf} \delta_\xi A_f \\ &= 0. \end{aligned} \quad (2.26)$$

Therefore we know that the last three terms in Eq. (2.24) also gives no contribution to  $K[\xi_1, \xi_2]$ .

By applying Eqs. (2.16), (2.17), (2.20), and (2.23), we can show that, at the horizon,  $\int_{\partial C} \epsilon_{abcd} e^{-2\alpha\phi} A_e \xi^e \nabla^a \chi^b \rightarrow 0$ . Hence, from Eq. (2.14) we find

$$\int_{\partial C} Q_{cd} = - \int_{\partial C} \epsilon_{abcd} \nabla^a \xi^b. \quad (2.27)$$

Denoting by  $\delta_\xi$  the variation corresponding to diffeomorphism generated by  $\xi$ , for the Noether current we have  $\delta_{\xi_2} \mathbf{J}[\xi_1] = d[\xi_2(\Theta[\phi, \mathcal{L}_{\xi_1} \phi] - \xi_1 \cdot \mathbf{L})]$ . Substituting it into Eq. (2.4) and using Eq. (2.12) we obtain

$$\begin{aligned} \delta_{\xi_2} H[\xi_1] &= \int_{\partial C} (\xi_2 \Theta[\phi, \mathcal{L}_{\xi_1} \phi] - \xi_1 \Theta[\phi, \mathcal{L}_{\xi_2} \phi] - \xi_2 \xi_1 \cdot \mathbf{L}) \\ &= \int_{\partial C} \epsilon_{abcd} [\xi_2^a \nabla_e (\nabla^e \xi_1^b - \nabla^b \xi_1^e) - \xi_1^a \nabla_e (\nabla^e \xi_2^b \\ &\quad - \nabla^b \xi_2^e)] - 4 \int_{\partial C} \epsilon_{abcd} e^{-2\alpha\phi} \{ \xi_2^a F^{fb} [F_{ef} \xi_1^e \\ &\quad + (\xi_1^e A_e)_{;f}] - \xi_1^a F^{fb} [F_{ef} \xi_2^e + (\xi_2^e A_e)_{;f}] \} \\ &\quad - \int_{\partial C} \epsilon_{abcd} [4 R_e^b (\xi_1^a \xi_2^e - \xi_2^a \xi_1^e) + \xi_2^a \xi_1^b \mathbf{L}] \\ &\quad - 4 \int_{\partial C} \epsilon_{abcd} (\xi_2^a \xi_1^e - \xi_1^a \xi_2^e) \nabla^b \phi \nabla_e \phi. \end{aligned} \quad (2.28)$$

At the horizon, applying Eqs. (2.15), (2.16), (2.17), and (2.19)–(2.23) we see that

$$\begin{aligned} &\int_{\partial C} \epsilon_{abcd} (\xi_2^a \xi_1^e - \xi_1^a \xi_2^e) \nabla^b \phi \nabla_e \phi \\ &= \int_{\partial C} \hat{\epsilon}_{cd} \left( \frac{1}{\kappa} \frac{\chi^2}{\rho^2} \right) \left[ \frac{|\chi|}{\rho} \rho_b \rho^e - \left( \frac{\rho}{|\chi|} + t \cdot \rho \right) \chi_b \chi^e \right] \\ &\quad \times (\mathcal{T}_2 D \mathcal{T}_1 - \mathcal{T}_1 D \mathcal{T}_2) \nabla^b \phi \nabla_e \phi \\ &= 0, \end{aligned} \quad (2.29)$$

$$\begin{aligned}
\int_{\partial C} \epsilon_{abcd} \xi_2^a \xi_1^b \mathbf{L} &= \int_{\partial C} \hat{\epsilon}_{cd} \mathbf{L} \left[ \frac{|\chi|}{\rho} \mathcal{T}_2 \rho_b + \left( \frac{\rho}{|\chi|} + t \cdot \rho \right) \mathcal{R}_2 \chi_b \right] \\
&\quad \times (\mathcal{T}_1 \chi^b + \mathcal{R}_1 \rho^b) \\
&= \int_{\partial C} \hat{\epsilon}_{cd} \mathbf{L} \left[ \frac{|\chi|}{\rho} \mathcal{T}_2 \mathcal{R}_1 \rho^2 \right. \\
&\quad \left. + \left( \frac{\rho}{|\chi|} + t \cdot \rho \right) \mathcal{R}_2 \mathcal{T}_1 \chi^2 \right] \\
&= 0,
\end{aligned} \tag{2.30}$$

and

$$\begin{aligned}
\int_{\partial C} \epsilon_{abcd} R_e^b (\xi_1^a \xi_2^e - \xi_2^a \xi_1^e) &= \int_{\partial C} \hat{\epsilon}_{cd} R_e^b \left( \frac{1}{\kappa} \frac{\chi^2}{\rho^2} \right) \left[ \frac{|\chi|}{\rho} \rho_b \rho^e \right. \\
&\quad \left. - \left( \frac{\rho}{|\chi|} + t \cdot \rho \right) \chi_b \chi^e \right] \\
&\quad \times (\mathcal{T}_1 D \mathcal{T}_2 - \mathcal{T}_2 D \mathcal{T}_1) \\
&= 0.
\end{aligned} \tag{2.31}$$

Substituting Eqs. (2.29), (2.26), (2.30), and (2.31) into Eq. (2.28) we find

$$\begin{aligned}
\delta_{\xi_2} H[\xi_1] &= \int_{\partial C} \epsilon_{abcd} [\xi_2^a \nabla_e (\nabla^e \xi_1^b - \nabla^b \xi_1^e) \\
&\quad - \xi_1^a \nabla_e (\nabla^e \xi_2^b - \nabla^b \xi_2^e)].
\end{aligned} \tag{2.32}$$

We can interpret the left side of Eq. (2.28), the variation of the boundary term  $J$ , since the ‘‘bulk’’ part of the generator  $H[\xi_1]$  on the left side vanishes on shell. On the other hand, the change in  $J[\xi_1]$  under a surface deformation generated by  $J[\xi_2]$  can be precisely described by Dirac bracket  $\{J[\xi_1], J[\xi_2]\}^*$  [8]. Thus we have

$$\begin{aligned}
\{J[\xi_1], J[\xi_2]\}^* &= \int_{\partial C} \epsilon_{abcd} [\xi_2^a \nabla_e (\nabla^e \xi_1^b - \nabla^b \xi_1^e) \\
&\quad - \xi_1^a \nabla_e (\nabla^e \xi_2^b - \nabla^b \xi_2^e)].
\end{aligned} \tag{2.33}$$

Inserting Eqs. (2.20), (2.21), and (2.23) into (2.33) we obtain

$$\begin{aligned}
\{J[\xi_1], J[\xi_2]\}^* &= - \int_{\partial C} \hat{\epsilon}_{cd} \left[ \frac{1}{\kappa} (\mathcal{T}_1 D^3 \mathcal{T}_2 - \mathcal{T}_2 D^3 \mathcal{T}_1) \right. \\
&\quad \left. - 2\kappa (\mathcal{T}_1 D \mathcal{T}_2 - \mathcal{T}_2 D \mathcal{T}_1) \right].
\end{aligned} \tag{2.34}$$

For any one-parameter group of diffeomorphism satisfying conditions (2.20) and (2.21), it is also easy to check that

$$\begin{aligned}
\{\xi_1, \xi_2\}^a &= (\mathcal{T}_1 D \mathcal{T}_2 - \mathcal{T}_2 D \mathcal{T}_1) \chi^a + \frac{1}{\kappa} \frac{\chi^2}{\rho^2} D(\mathcal{T}_1 D \mathcal{T}_2 \\
&\quad - \mathcal{T}_2 D \mathcal{T}_1) \rho^a.
\end{aligned} \tag{2.35}$$

The Hamiltonian (2.6) consists of two terms, but Eqs. (2.29) and (2.26) and discussion about  $\xi \cdot \Theta$  in Ref. [8] show that the second terms make no contribution. Then, we have

$$\begin{aligned}
J[\{\xi_1, \xi_2\}] &= \int_{\partial C} \hat{\epsilon}_{cd} \left[ 2\kappa (\mathcal{T}_1 D \mathcal{T}_2 - \mathcal{T}_2 D \mathcal{T}_1) \right. \\
&\quad \left. - \frac{1}{\kappa} D(\mathcal{T}_1 D^2 \mathcal{T}_2 - \mathcal{T}_2 D^2 \mathcal{T}_1) \right].
\end{aligned} \tag{2.36}$$

On shell Eq. (2.7) can be expressed as

$$\{J[\xi_1], J[\xi_2]\}^* = J[\{\xi_1, \xi_2\}] + K[\xi_1, \xi_2]. \tag{2.37}$$

Therefore, we know that from Eqs. (2.34) and (2.36) the central term is

$$K[\xi_1, \xi_2] = \int_{\partial C} \hat{\epsilon}_{cd} \frac{1}{\kappa} (D \mathcal{T}_1 D^2 \mathcal{T}_2 - D \mathcal{T}_2 D^2 \mathcal{T}_1). \tag{2.38}$$

It is interesting to note that the constraint algebra (2.37) with Eqs. (2.34), (2.36), and (2.38) has same form as that for the vacuum case [8]. In the next section, we will study statistical-mechanical entropies of some stationary dilaton black holes by using the constraint algebra and conformal field theory methods.

### III. STATISTICAL ENTROPY OF STATIONARY DILATON BLACK HOLE

In order to construct a standard Virasoro subalgebra from constraint algebra (2.34) and (2.36)-(2.38), as Cadoni, Mignemi, and Carlip did in Refs. [27,8] we define a new generator  $\int dv J$  in which the function  $v$  takes period  $T$ . From stationary conditions (2.17) we know that a one-parameter group of diffeomorphism satisfying Eqs. (2.22) and (2.35) can be taken as

$$\mathcal{T}_n = \frac{T}{2\pi} \exp \left[ in \left( \frac{2\pi}{T} v + C_\alpha (\varphi - \Omega_H v) \right) \right], \tag{3.1}$$

where  $C_\alpha$  is an arbitrary constant. We should note that one-parameter group (3.1) is also valid for the static black hole since it is a special case of the stationary black hole with  $\Omega_H = 0$ . Substituting Eq. (3.1) into central term (2.38) and using condition (2.22) we obtain

$$K[\mathcal{T}_m, \mathcal{T}_n] = - \frac{i A_H}{8\pi} \frac{2\pi}{\kappa T} m^3 \delta_{m+n,0}, \tag{3.2}$$

where  $A_H = \int_{\partial C} \hat{\epsilon}_{cd}$  is the area of the event horizon. Equation (2.37) thus takes the standard form of a Virasoro algebra

$$i\{J[\mathcal{T}_m], J[\mathcal{T}_n]\} = (m-n)J[\mathcal{T}_{m+n}] + \frac{c}{12} m^3 \delta_{m+n,0}, \tag{3.3}$$

with central charge

$$\frac{c}{12} = \frac{A_H}{8\pi} \frac{2\pi}{\kappa T}. \quad (3.4)$$

The boundary term  $J[\mathcal{T}_0]$  can easily be obtained by using Eqs. (2.3), (2.14), and (3.1), which is given by

$$J[\mathcal{T}_0] = \Delta = \frac{A_H}{8\pi} \frac{\kappa T}{2\pi}. \quad (3.5)$$

From standard Cardy's formula [8]

$$\rho(\Delta) \sim \exp \left\{ 2\pi \sqrt{\frac{c}{6} \left( \Delta - \frac{c}{24} \right)} \right\}, \quad (3.6)$$

we know that the number of states with a given eigenvalue  $\Delta$  of  $J[\mathcal{T}_0]$  grows asymptotically for large  $\Delta$  as

$$\rho(\Delta) \sim \exp \left[ \frac{A_H}{4} \sqrt{2 - \left( \frac{2\pi}{\kappa T} \right)^2} \right]. \quad (3.7)$$

Only if we take the period  $T$  as the periodicity of the Euclidean black hole, i.e.,

$$T = \frac{2\pi}{\kappa}, \quad (3.8)$$

the statistical entropy of the stationary dilaton black hole

$$S_0 \sim \ln \rho(\Delta) = \frac{A_H}{4}, \quad (3.9)$$

coincides with the standard Bekenstein-Hawking entropy.

#### IV. LOGARITHMIC CORRECTIONS TO BLACK HOLE ENTROPY

Now let us consider the first-order quantum correction to the entropy. In order to do that, we should first derive the logarithmic corrections to the Cardy formula.

In Refs. [28,17], Carlip showed that the number of states is

$$\rho(\Delta) = \int d\tau e^{-2\pi i \Delta \tau} e^{-2\pi i \Delta_0 (1/\tau)} e^{(2\pi i c/24) \tau} e^{2(\pi i c/24) (1/\tau)} \times \tilde{Z}(-1/\tau), \quad (4.1)$$

where  $\tilde{Z}(-1/\tau)$  approaches to constant,  $\rho(\Delta_0)$ , for large  $\tau$ . So the integral (4.1) can be evaluated by a steep descent provided that the imaginary part of  $\tau$  is large at the saddle point.

The integral takes the form

$$I[a, b] = \int d\tau e^{2\pi i a \tau + 2\pi i b/\tau} f(\tau). \quad (4.2)$$

The argument of the exponent is extremal at  $\tau_0 = \sqrt{b/a}$ , and expanding around  $\tau_0$ , one has [17]

$$I[a, b] \approx \int d\tau e^{4\pi i a \sqrt{ab} + 2\pi i b/\tau_0^3 (\tau - \tau_0)^2} f(\tau_0) = \left( -\frac{b}{4a^3} \right)^{1/4} e^{4\pi i \sqrt{ab}}. \quad (4.3)$$

Comparing Eqs. (4.1) with (4.2) we know

$$a = \frac{c}{24} - \Delta, \quad b = \frac{c}{24} - \Delta_0. \quad (4.4)$$

Therefore, for large  $\Delta$ , if we let  $c_{\text{eff}} = c - 24\Delta_0$ , the number of states can be expressed as

$$\rho_{cq}(\Delta) \approx \left[ \frac{c_{\text{eff}}}{96 \left( \Delta - \frac{c}{24} \right)^3} \right]^{1/4} \times \exp \left\{ 2\pi \sqrt{\frac{c_{\text{eff}}}{6} \left( \Delta - \frac{c}{24} \right)} \right\} \rho(\Delta_0). \quad (4.5)$$

The exponential part in (4.5) gives the Carlip's result (C.3) in Appendix C in Ref. [8], the factor before the exponent devotes the logarithmic correction to black hole entropy.

By Using the central charge (3.4), eigenvalue (3.5), constraint condition of the period (3.8), and new Cardy formula (4.5), we know that the statistical entropy including first-order quantum correction is given by

$$S = \frac{A_H}{4} - \frac{3}{2} \ln \frac{A_H}{4} + \ln c + \text{const.},$$

$$= \frac{A_H}{4} - \frac{1}{2} \ln \frac{A_H}{4} + \text{const.} \quad (4.6)$$

The first line has two logarithmic terms and agrees with Carlip's results (1.2) [17]. However, after we take  $T = 2\pi/\kappa$ , the second shows that the factor of the logarithmic term becomes  $-\frac{1}{2}$ , which is different from Kaul and Majumdar's result  $-\frac{3}{2}$ .

#### V. SUMMARY AND DISCUSSION

We extend Carlip's investigation in Ref. [8] to four dimensional low-energy string theory by the covariant phase techniques. With Carlip's boundary conditions, a standard Virasoro subalgebra with corresponding central charge for stationary dilaton black hole is constructed at a Killing horizon. We find that only we take  $T$  as the periodicity of the Euclidean black hole,  $T = 2\pi/\kappa$ , the statistical entropy of the stationary dilaton black hole yielded by standard Cardy formula agrees with its Bekenstein-Hawking entropy. Therefore, Carlip's conclusion—the asymptotic behavior of the density of states may be determined by the algebra of diffeomorphism at horizon—is valid for stationary dilaton black holes obtained from the low-energy effective field theory with Lagrangian (2.8).

When we consider first-order quantum correction the entropy contains extra logarithmic terms which agree with Car-



lip's results (1.2) [17]. However, from above discussions we know that in order to get the Bekenstein-Hawking entropy we have to take  $T=2\pi/\kappa$ . That is to say, we cannot set central charge  $c$  to be a universal constant, independent of area of the event horizon, by adjusting periodicity  $T$  as Carlip suggested in Ref. [17]. Therefore, the factor of the logarithmic term is  $-\frac{1}{2}$ , which is different from Kaul and Majumdar's result,  $-\frac{3}{2}$ .

From the derivation given in the Sec. IV we know that the new Cardy formula (4.5) is valid for general black hole whether or not the black hole is dilatonic. Hence, the factor of the logarithmic term will be  $-\frac{1}{2}$  as long as the spacetime is such that (2.38) obeys. This means that the discrepancy

between Carlip's [17] approach and that of Kaul and Majumdar [16] is not just for the dilaton black hole, but for any black hole which respects (2.38), where  $T$  is the periodicity of the Euclidean black hole.

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